Bk. 1

COMPUTATIONS
+
Early PROGRAM
EVOLUTION

W. RALL

RECORD

7530-222-3525
FEDERAL SUPPLY SERVICE
(GPO)

W. Rall

National Institutes of Health
Bethesda, Maryland

Bldg 10, 11-N-119

Computer Programming
&
Computation Record

This record begun May 10, 1963,
also summarizes loose records
dating back to early 1959

Compartmental computations with Mones' program are
in separate book (BK. 2)

Gross Summary

1959 early explorations with Ezra on IBM-650
FORTRANSIT

1960 productive summer with Jeanne & Ezra
computing equipotential contours

late 1960
early 1961 Some additional runs & reprogramming for 650
Also "Dendritic Current & Potential" with Brunelle

1962 Programming for Honeywell 800
- WXR101C Soma Transients for Soma Current Step [ρ, L]
- WXR50$\overset{3}{1}$C Equipotential Contours for axial weights
- WXR60$\overset{2}{1}$C Extrapolation of Dendritic Branching
- WXR546C Extracellular Step & Spike rel. to Electrotonus

late 1962, early 1963: used Mone's program: see separate book

1959

Ezra Shahn devised preliminary FORTRANSIT programs for IBM-650

Program #

May 1959 Legendre Polynomials — 13

March 1959 Isopotentials - Superposition — 10.1
June 1959 Superposition ——— 10.6
Most of this time dogged by faulty printer.

Feb 1960 Soma - Dendritic Field No. 19.1
July 1960 " " " 19.2

$$V = \sum_{I=1}^{n} W(I) \left[\frac{1}{\sqrt{R^2 + X^2 - RX\cos\theta}} - \frac{1}{R} + \sum_{J=1}^{M} \left(\frac{1}{RX}\right)^{J+1} \left(\frac{J}{J+1}\right) P(J) \right]$$

April 1960 Cosine Table Maker No. 20.1

March 1960 Dendritic Superposition No. 21.1
May 1960 " " 21.2
" " " " 21.3 ~~A~~ B
April 1960 Interpolation No. 22.1
June 1960 Isopot. Interpolation No. 23.1
July 1960 Axonal Linear Combination No. 24.1
" " Dendrite - Trunk " 25.1
Aug 1960 Dendritic Branch Superposition 26.1

COMPUTATION PROCEDURES

(The following first draft describing the computation procedures used by Rall, Shahn and Altmann for extracellular potential fields of neurons was written by Ezra Shahn in October, 1960. The program numbers refer to "FORTRANSIT" programs that were also written by Shahn; this introductory note added by W. Rall.)

The machine calculations for the neuron fields were all performed on an IBM 650. It was assumed that the field about any specified configuration of dendrites could be obtained by a superposition of all the single dendrite solutions, which in turn were assumed to be of the same type. This set of assumptions was used to simplify the programming in the following way. The field for one dendrite, which was axially symmetric, was calculated from the equation

$$V(R,\theta) = \sum_{I=1}^{N} W(I)\Big[\Big(\frac{1}{\sqrt{R + x - 2Rx\cos\theta}} - \frac{1}{R}\Big) + \sum_{J=1}^{M} \Big(\frac{1}{Rx}\Big)^{J+1} \frac{J}{J+1} P(J)\Big]$$

where N is the number of subdivisions made along the dendrite, the W(I) are the weights attached to each such subdivision, x is the distance of the center of the I^{th} subdivision from the center of the soma, P(J) are Legendre polynomials and R and θ are the coordinates of the field point. M was selected to be large enough to insure convergence of the last sum, and was set at 10. For small angles N was set at 160, for larger angles at 80. The values of W(I) were hand calculated and considered as input information.

[The program which accomplished this is called No. 19.2. Its input consists of : one card having in the first four fields (values of Dx, the distance between subdivisions along the dendrite) (NPROB, an arbitrary problem number) (M, see above,) and (CONST, an arbitrary number used to terminate the reading of the W(I)); a card having in the first three fields THMIN, THMAX, and DTH, which represent the minimum, maximum angles desired, and the size of the steps between them in radians; a similar card for RMIN, RMAX, DR; the set of W(I), one per card in the first field. The output has the first three input cards reproduced, followed by the calculated values of V, R, θ, in the first three fields in that order. For 160 weights, and R = 1, 20 in steps of 1 the running time is 1/2 hour per angle.]

Since for a given R the V as a function of θ is fairly smooth it is not necessary to use program 19.2 for all angles. Typically, if half degree steps are desired, 35 or so angles are all that are needed for adequate determination of V, rather than 360. These are spaced closely together at i.e for small θ first, and quite far apart towards the end, the intermediate values are obtained by a straight-forward linear interpolation. The program which does this is called 22.1.

[Program 22.1 interpolates to half degree steps the output of program 19.2 <u>provided</u> that the last angle of the set be greater than 3.1 radians. Before this program can be used the output of 19.2 must be sorted into groups of equal R each of which is ordered by θ (increasing). These are then completely arranged to serve as the input for 22.1.

The output will consist of 360 cards for each R with θ increasing by half degree each. The first three fields of each card have V, R, θ, θ in radians.]

This "raw data" forms the basis of all subsequent superpositions which are done with programs 21.3A and 21.3B. A problem is fully specified when the angular coordinates of the dendrites are given with reference to the coordinate system in which one wants to describe the field. Program 21.3A is written so that each of the field points which are desired is located by its angular displacement from each of the dendrite locations. These values are then translated into integers representing the number of half degrees which is closest to the angle in question. These numbers are punched sequentially 7 per card, and are referred to as the ICOSA table. They serve as part of the input for 21.3B.

[The initial input of 21.3A is a table of integers equal to the truncated values 10000 cosØ where Ø ranges from $1/2^{\circ}$ to 90° in $1/2^{\circ}$ steps. These 180 values are the six per card output of program 20.1 which is specifically for the purpose of generating this table. These 30 cards are followed by one having in the first 3 fields (NPROB, an arbitrary problem number) (N, the number of dendrites) (RMAX, any number as it's not used); a card having in the first 6 fields the values of THMIN, THMAX, DTH, PHMIN, PHMAX, DPH, as the minimum, maximum and step size of the θ-Ø grid of field points, the number of which times N cannot exceed 999; N cards each having in the first two fields the pair of values of θ and Ø that specifies the position of a dendrite. These may be in any order. The output, as mentioned above gives the set of values corresponding to the angular positions of each of the field points with respect to each of the dendrites, these values being in number of half degrees. These are 7 per card. They are preceded by the first two cards of the input, which must be removed before use.]

The desired superposition can now proceed without further delay, and, because of the separation of the steps into programs 19.2 - 22.1 and 21.3A, with quite good speed. This is done by the procedure of program 21.3B, which requires as input the specification of the field point grid, a deck of cards for each R value that is of interest, and of course the ICOSA deck just described. The procedure is then to form a sum for the value of the potential at each field point, this sum consisting of the values of the field for a single dendrite corresponding to the position that each field point has with relation to each dendrite. The output is then of the form V, θ, ∅, R, NPROB for each field point θ, ∅ in <u>degrees</u>.

[Input requirements for 21.3B are: a card giving (NPROB, an arbitrary problem number) (N, the number of dendrites) (RMAX, the largest R-value to be considered) (NFP D, the number of field points times dendrites (less than 999)); a card giving THMIN, THMAX, DTH, PHMIN, PHMAX, DPH for field points, which must be identical with the corresponding card for 21.3A, the ICOSA deck output of 21.3A, and a deck of 360 cards for each R desired, these decks being the output of 22.1. The output of 21.3B repeats the first two cards, which may be discarded after identification is made, and the values V, θ, ∅, R, NPROB in the first 5 fields. θ and ∅ are here in degrees.]

In order that these dendritic superpositions might more easily be plotted, a program No. 23.1 was written which takes the output from 21.3B, sorted for constant θ - ∅, i.e. for given angle, and R increasing, and searches for those R values having potentials which bracket a desired isopotential. This is repeated for as many isopotentials as one pleases up to 50, and the entire process iterates for each set defined by a given θ,∅. The output is given as x, θ and ∅, RINT, NPROB, where x is the isopotential and RINT is the interpolated R-value. Since these come for constant θ - ∅, sorting of the entire output by x might be useful. θ and ∅ are in degrees.

[Input requirements for 23.1 are: a card giving (NPROB, an arbitrary problem number) (ISOP, the number of isopotentials sought) (NRAD, the number of radius values for each θ - Ø set); ISOP cards each having in the first field the value of an isopotential being sought; NRAD cards having V, θ, Ø, R for each R, this set repeated for each θ - Ø pair. The first card of the output reproduces the first card of the input. The remainder is described above.]

Note that the input requirement of 23.1 and the output of 22.1 are incompatible--that is, the isopotential interpolation program cannot be used to give a set of values for the field of the single dendrite. However, if the output of 22.1 is duplicated in such a fashion that the R values are moved from the third to the fourth fields, then this duplicated deck can be used as input for 23.1, and easily plotted results are obtained.

To find the effect of an axon on the field of a set of dendrites another superposition is possible. With a different set of weights programs 19.2 and 22.1 can be used to generate the field due to an axon. This can then be added to the field of a set of dendrites, in such a way that if VP' is the original dendrite field value, V the axon value for supplementary θ, and A and B are constants used to satisfy boundary conditions, the combined field, VP, is

$$VP = A(VP' - BV).$$

This is done by program 24.1.

[Input requirements for 24.1 are: a card with (NPROB, an arbitrary problem number) (A and B, constants, see above), and (NFP, the number of field points); 360 cards, output of 22.1, having in their first field the values of the field of an axon for a given R as a function of θ; NFP cards, the output of 21.3B, for the same R as the axonal field. These last two sets, axonal field and dendritic field must alternate for each R. Color coding is clearly useful in this context. The output of 24.1 repeats the first card and follows with the combined values VP, θ, ∅, R, NPROB for each field point.]

It is clear that for plotting purposes it is desirable to use the output of 24.1 as input for 23.1 and thus obtain interpolated values of R to correspond to a previously selected set of isopotentials. Attention is again called to the necessity of sorting at this point.

A dendrite-field altered by a change in the characteristics of the trunk can be achieved by superimposing the field of a single dendrite and that of a trunk, i.e. a shorter dendrite with suitably selected weights. These may be combined directly as the outputs of 19.2 and the result then fed into 22.1, 21.3A, 21.3B and 23.1 as a regular problem. Program 25.1 does this point by point addition, and has an output similar to that of 19.2, and is used as in 19.2

[Input requirements for 25.1 are: a card with (NPROB, an arbitrary problem number) (A, a constant normalizing factor) and (N, the number of R values for which potentials were calculated for each θ)-- this must be the same for dendrite and trunk; N cards having potentials in the first field for trunk calculations; N cards having potentials in the first field for dendrite calculations, these must be in the same R order as the trunk set; these two sets alternate for each θ. The first output card reproduces the first input card. The remaining output cards have VP, the normalized combined potential, R and θ in the first three fields, a blank, and NPROB.]

In the above all quantities starting with I, J, K, L, M, N are fixed point integers, the rest are floating point numbers.

Computations Completed Summer 1960
with Jeanne Altmann & Ezra Shahn

		Problem #
$(-S, +D_1)$	soma sink with single dendrite (near field, R=1 to 20)	100
	(far field, R=25 to 75	101-103
$(-S, +D_7)$	soma sink with seven dendrites (near field, R=1 to 20)	12-15
	(far field, R=25 to 75)	105, 106
$(-S, +A)$	~~Soma~~ sink with short axon source (near field, R=1, 20)	20
$(-A, +SD_7)$	axon sink with seven dendrites	40, 41
$(-S, +T)$	soma sink with trunk source	25
$(-ST, +D_1)$	soma-trunk sink with single dendrite	50
$(-ST_7, +D_7)$	soma-trunk sink with seven dendrites	51, 52
$(-S, +D_7)$	for $\varphi = 5^\circ, 10^\circ, 30^\circ$	16

Plotting was done on Maneuvering Board
(i.e. polar graph paper of 10" radius)

Ink drawings were made on tracing paper, of:

$-S, +D_1$ near field; far field

$-S, +D_7$ { near field; far field
also $\varphi = 0, 5^\circ, 10^\circ, 30^\circ$

$-A, +SD_7$ near field

$-ST_7 + D_7$ near field

Dimpled pear drawing

Axis orientation drawing.

Also my hand calculation & drawing of dimple vs branch number.

late 1960

My own Fortransit modifications & extensions tested out with help of John Witmer.

External Field — #101

Internal-External Field for Source Sink Pair #1001
(9/23/60)

1002

1050